Chapter 1: Continuity.

Q1 Examine the continuity of the following function at given points.

i) $$f(x) = \begin{cases} \dfrac{\sin 2x}{\sqrt{1-\cos 2x}} & \text{for } 0 < x < \pi/2 \\ \dfrac{\cos x}{\pi - 2x} & \text{for } \pi/2 < x < \pi \end{cases} \quad \text{at } x = \pi/2$$

ii) $$f(x) = \begin{cases} \dfrac{1-\sin x}{\left(\frac{\pi}{2}-x\right)^2} & \text{for } x \neq \pi/2 \\ 3 & \text{for } x = \pi/2 \end{cases} \quad \text{at } x = \pi/2$$

iii) $$f(x) = \begin{cases} \dfrac{10^x + 7^x - 14^x - 5^x}{1-\cos 4x} & \text{for } x \neq 0 \\ \dfrac{10}{7} & \text{for } x = 0 \end{cases} \quad \text{at } x = 0$$

Q2 Discuss the continuity of the following function, which of these functions have a removable discontinuity? Redefine the function so as to remove the discontinuity.

i) $$f(x) = \begin{cases} \dfrac{(e^{2x}-1)\tan x}{x \sin x} & \text{for } x \neq 0 \\ e^2 & \text{for } x = 0 \end{cases} \quad \text{at } x = 0$$

ii) $$f(x) = \begin{cases} \dfrac{1-\sin x}{(\pi - 2x)^2} & \text{for } x \neq \pi/2 \\ \dfrac{2}{7} & \text{for } x = \pi/2 \end{cases} \quad \text{at } x = \pi/2$$

Q3 If $$f(x) = \begin{cases} \dfrac{x^2-9}{x-3} + \alpha & \text{for } x > 3 \\ 5 & \text{for } x = 3 \\ 2x^2 + 3x + \beta & \text{for } x < 3 \end{cases}$$ is continuous at $x = 3$

then find 'α' and 'β'.

Chapter 2 : Differentiation.

Q1 If $y = \tan^{-1}\left[\sqrt{\frac{1+\cos x}{1-\cos x}}\right]$ find $\frac{dy}{dx}$.

Q2 If $y = \tan^{-1}\left[\frac{x}{1+6x^2}\right]$ find $\frac{dy}{dx}$.

Q3 If $y = \sqrt{\frac{(x^3-3x+2)^5}{(x^2+4x-5)^3}}$ find $\frac{dy}{dx}$.

Q4 If $\cos(xy) = \sin(x+y)$ find dy/dx.

Q5. If $x = a\sec^3\theta$ and $y = a\tan^3\theta$ find $\frac{dy}{dx}$ at $\theta = \frac{\pi}{3}$.

Q6 If $x^2+y^2 = t - \frac{1}{t}$ and $x^4+y^4 = t^2+\frac{1}{t^2}$

show that $\frac{dy}{dx} = \frac{1}{x^3y}$.

Q7 Differentiate $\cos^{-1}(\sin x)$ with respect to $\tan^{-1}x$.

Q8 Differentiate $(\cos x)^{\sin x}$ w.r.t. $(\sin x)^{\cos x}$

Q9 find $\frac{d^2y}{dx^2}$ if $x = a\cos^3\theta$ and $y = a\sin^3\theta$.

Q10 find $\frac{d^2y}{dx^2}$ if $y = x\cdot\log x$.

Q11 find $\frac{d^2y}{dx^2}$ if $y = \frac{x}{\log x}$.

Q12 If $y = \tan^{-1}\left(\frac{5x+1}{3-x-6x^2}\right)$ then find $\frac{dy}{dx}$.

Q13 If $y = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ find $\frac{dy}{dx}$.

Q14 If $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ find $\frac{dy}{dx}$.

Q15 If $y = \cos^{-1}\left(\frac{3\sin x + 4\cos x}{5}\right)$ find $\frac{dy}{dx}$.

Q16 If $y = \tan^{-1}(\sec x + \tan x)$ find $\frac{dy}{dx}$.

Q17 If $y = x^{x^{2x}}$ find dy/dx

Chapter 3: Applications of Derivatives

Q1 find the equations of Tangent & Normal to the curve $y = 3x^2 - x + 1$ at $(1, 3)$.

Q2 find the equations of Tangent & Normal to the curve $y = 6 - x^2$ where the normal is parallel to the line $x - 4y + 3 = 0$

Q3 find the approximate value of $\sqrt[3]{27.027}$

Q4 find the approximate value of $e^{1.005}$

Q5 Verify Rolle's Theorem for $f(x) = x^2 - 5x + 9$, $x \in [1, 4]$.

Q6 Verify LMVT for $f(x) = x^2 - 3x - 1$ $x \in \left[-\frac{11}{7}, \frac{13}{7}\right]$

Q7 find the maximum & minimum value of function $f(x) = 3x^3 - 9x^2 - 27x + 15$.

Q8 A rod of 108 meters long is bent to form a rectangle. find its dimension if the area is maximum.

Q9 An open box is to be made out of a piece of square card board of side 18 cms by cutting off equal squares from the corners & turning up the sides. find the maximum volume of box.

Chapter 4: Integration

Q1 $\int x^a + a^a + a^x \, dx$

Q2 $\int e^x (1 + \tan x + \tan^2 x) \, dx$

Q3 Prove that $\int \tan x \, dx = \log|\sec x| + c$.

Q4 $\int \frac{\sin\sqrt{x}}{\sqrt{x}} \, dx$

Q5 $\int \frac{\sin(x+a)}{\cos(x-b)} \, dx$

Q6 $\int \frac{3e^x + 4}{2e^x - 8} \, dx$

Q7 $\int \frac{1}{1 + x - x^2} \, dx$

Q8 $\int \frac{1}{x^2 - 5x + 6} \, dx$

Q9 $\int \frac{1}{\sqrt{3 + 4x - 4x^2}} \, dx$

Q10 $\int \frac{1}{1 + 3\sin^2 x + 8\cos^2 x} \, dx$

Q11 $\int \frac{1}{4 + 5\sin x} \, dx$

Q12 $\int \frac{1}{3 - 2\cos 2x} \, dx$

Q13 $\int x \cdot \tan^2 x \, dx$

Q14 $\int \frac{(3x-2)}{(x+1)^2(x+3)}\,dx$

Q15 $\int \frac{1}{x \cdot \log x}\,dx$

Q16 $\int \tan^{-1}\left[\sqrt{\frac{1-\sin x}{1+\sin x}}\right] dx$

Q17 $\int \cot^{-1}\left[\frac{1+\sin x}{\cos x}\right] dx$

Q18 $\int \sqrt{\tan x}\,dx$

Q19 $\int \sqrt{x^2+x+1}\,dx$

Q20 $\int (3x-2)\sqrt{x^2+x+1}\,dx$

Chapter 5: Definite Integral:

Q1 $\int_0^{\pi/4} \tan^2 x\,dx$

Q2 $\int_0^{\pi/4} \sqrt{1+\sin 2x}\,dx$

Q3 $\int_0^{\pi/2} \cos^3 x\,dx$

Q4 $\int_0^{\pi/2} \frac{1}{5+4\cos x}\,dx$

Q5 $\int_0^{1} \sin^{-1}\left(\frac{2x}{1+x^2}\right) dx$

Q6 $\int_0^{\pi/2} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$

Q7 $\int_0^{\pi/4} \log(1+\tan x)\,dx$

Q8 $\int_2^{7} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{9-x}}\,dx$

Q9 $\int_4^{7} \frac{(11-x)^2}{x^3+(11-x)^2}\,dx$

Q10 $\int_0^{\pi} x \cdot \sin^2 x\,dx$.

Q11 $\int_{-\pi/4}^{\pi/4} x^3 \cdot \sin^4 x\,dx$.

Q12 $\int_{-\pi/2}^{\pi/2} \sin^2 x\,dx$

Chapter 6: Applications of Definite Integral.

Q1 find the area of circle $x^2+y^2=25$ by integration.

Q2 find the area of ellipse $\frac{x^2}{4}+\frac{y^2}{25}=1$ by integration.

Q3 find area of region lying between the parabolar $y^2=x$ and $x^2=y$

Q4 find the area enclosed between the circle $x^2+y^2=16$ and $y^2=6x$.

Q5. find the area of the region bounded by the curves $x^2=16y$ $y=1$, $y=4$ & the y-axis lying in first quadrant.

Q6 find the area of the region bounded by $y=x^2$, $x=1$, $x=3$ & x-axis.

Chapter 7: Differential Equations.

Q1 Determine order & degree of the following D.F.

a) $\frac{d^4y}{dx^4}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^3$

b] $\sqrt{1+\frac{1}{\left(\frac{dy}{dx}\right)^2}}=\left(\frac{d^2y}{dx^2}\right)^{3/2}$

Q2 Verify that $y=ae^x+be^{-2x}$ $a,b\in R$ is a solution of the differential equation $\frac{d^2y}{dx^2}+\frac{dy}{dx}=2y$.

Q3 Verify that $y=c_1\sin x+c_2\cos x$ $c_1, c_2 \in R$ is a solution of the differential equation $\frac{d^2y}{dx^2}+y=0$

Q4 obtain the differential equation from $y=Ae^{3x}+Be^{-3x}$.

Q5 Solve: $\frac{dy}{dx}=\cos(x+y)$

Q6 $(x^2+y^2)\,dx-2xy\,dy=0$ solve.

Q7. $\cos^2x\cdot\frac{dy}{dx}+y=\tan x$

Q8 The rate of growth of bacteria is proportional to the number present. If initially there were 1000 bacteria and the number doubles in 1 hour, find the number of bacteria after $2\frac{1}{2}$ hours.

Q9 The rate of decay of certain substance is directly proportional to the amount present at that instant. Initially there are 27 gms of certain substance and three hours later it is found that 8 gms are left. find the amount left after one more hour.

Q10. A body cools according to Newton's law from $100^\circ C$ to $60^\circ C$ in 20 minutes. The temperature of surroundings being $20^\circ C$. How long will it take to cool down to $80^\circ C$.

Chapter 8 : Probability Distribution.

Q1 A r.v. X has the following probability distribution.

$X=x$	-2	-1	0	1	2	3
$p(x)$	0.1	K	0.2	$2K$	0.3	K.

Q2 find the variance & standard deviation of the random variable X whose probability distribution is given below.

x	0	1	2	3
$P[X=x]$	$1/8$	$3/8$	$3/8$	$1/8$

Q3 The p.d.f. of continuous random variable is given by

$$f(x) = \frac{x}{8} \quad 0 < x < 4$$
$$= 0 \quad \text{otherwise}$$

find i) $P(X \leq 2)$ ii) $P[2 < X \leq 3]$ iii) $P(X \geq 3)$

Q4 for the following probability distribution of X,

X	0	1	2	3	4
$P(X=x)$	K	$2K$	$4K$	$2K$	K

i) find value of K

ii) $P(X \geq 2)$, $P(X < 3)$, $P(X \leq 1)$

Chapter 9 : Binomial Distribution

Q1 Given $X \sim B(n, p)$

i) If $n = 10$ & $p = 0.4$ find $E(X)$ & $Var(X)$

ii) If $E(X) = 6$ $Var(X) = 4.2$ find n and p.

iii) If $n = 25$ $E(X) = 10$ find p and $SD(X)$

Q2 Suppose that 80% of the families own a television set If 10 families are interviewed at random find the probability that

i) seven families own a television set.

ii) at most three families own a television set.

Q3 A radar complex consists of eight units that operate independently. The probability that a unit detects an incoming missile is 0.9. find the probability that an incoming missile will

i) not be detected by any unit.

ii) be detected by atmost four units.